Diversification, Volatility, and Liquidity

NYC QWAFAFEW, November 17, 2010

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Summary

- ***True*** diversification is arguably the best way to manage risk
  - How do we measure diversification?
- In highly concentrated markets portfolio construction and optimization face severe challenges, need to be careful
  - Robust optimization techniques
- Highly concentrated markets are vulnerable to liquidity shocks
  - Need to model this vulnerability

Diversification

- *Nominal* diversification may not produce any significant benefits
  - US and international stocks
  - Mortgages from Florida and Arizona
  - Various hedge funds
- *True* diversification is much harder to achieve
- Present day markets: risk on/risk off

“On a "risk on" day, the mood of investors is confident and they flood into stocks and other investments perceived as risky, such as junk bonds, emerging markets and commodities. But when it's "risk off," money comes sloshing out of those investments and into so-called safe-haven investments such as U.S. Treasuries, the U.S. dollar or Japanese yen.”

'Macro' Forces in Market Confound Stock Pickers**,** WSJ, September 24, 2010

Diversification Measures

- In “Diversification, Volatilities and the Supercurrency: The FX Markets Since Bretton Woods” we review diversification measures http://ssrn.com/abstract=1607722 (MG and Greg Jones, 2010)

- (Absolute) realized or implied correlation:
  - 1 in a perfectly concentrated market
  - 0 in a perfectly diversified market
- Absorption ratio of Kritzman, Li, Page, and Rigobon (2010)
  - 1 in a perfectly concentrated market
  - 1/(the nominal number of assets) in a perfectly diversified market
- Herfindahl-Hirschman Index
  - 1 in a perfectly concentrated market
  - 1/(the nominal number of assets) in a perfectly diversified market

Diversification Measures (cont)

- Condition number of the covaraince matrix
  - 1 in a perfectly diversified market
  - very high in a perfectly concentrated market
- Effective dimension
  - the nominal number of assets in a perfectly diversified market
  - 0 in a perfectly concentrated market
- **Effective number = exp(Shannon entropy of the distribution)**
  - the nominal number of assets in a perfectly diversified market
  - 1 in a perfectly concentrated market

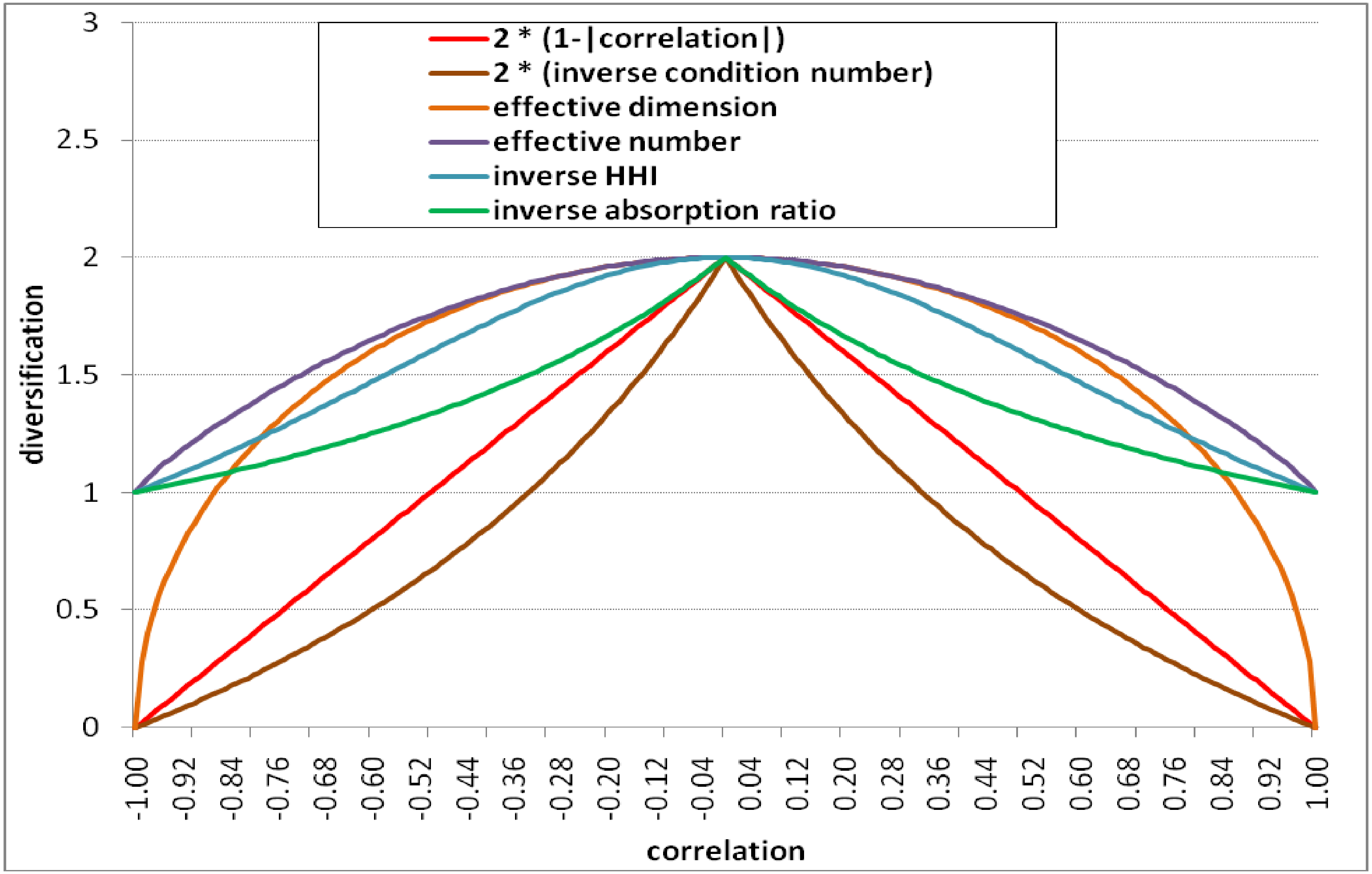
$$\Sigma = Q\mathrm{diag}(\theta_1^2, \theta_2^2, \ldots, \theta_N^2)Q', \quad \theta_1^2 \geq \theta_2^2 \geq \ldots \geq \theta_N^2$$

$$p_i = \frac{\theta_i^2}{\sum_k \theta_k^2} = \frac{\theta_i^2}{\mathrm{tr}\,\Sigma} \qquad \mathrm{erank}\,\Sigma = \exp\left(-\sum_{i=1}^{N} p_i \log p_i\right) \qquad \mathrm{edim}\,\Sigma = N\,\frac{(\det \Sigma)^{1/N}}{\mathrm{tr}\,\Sigma / N}$$

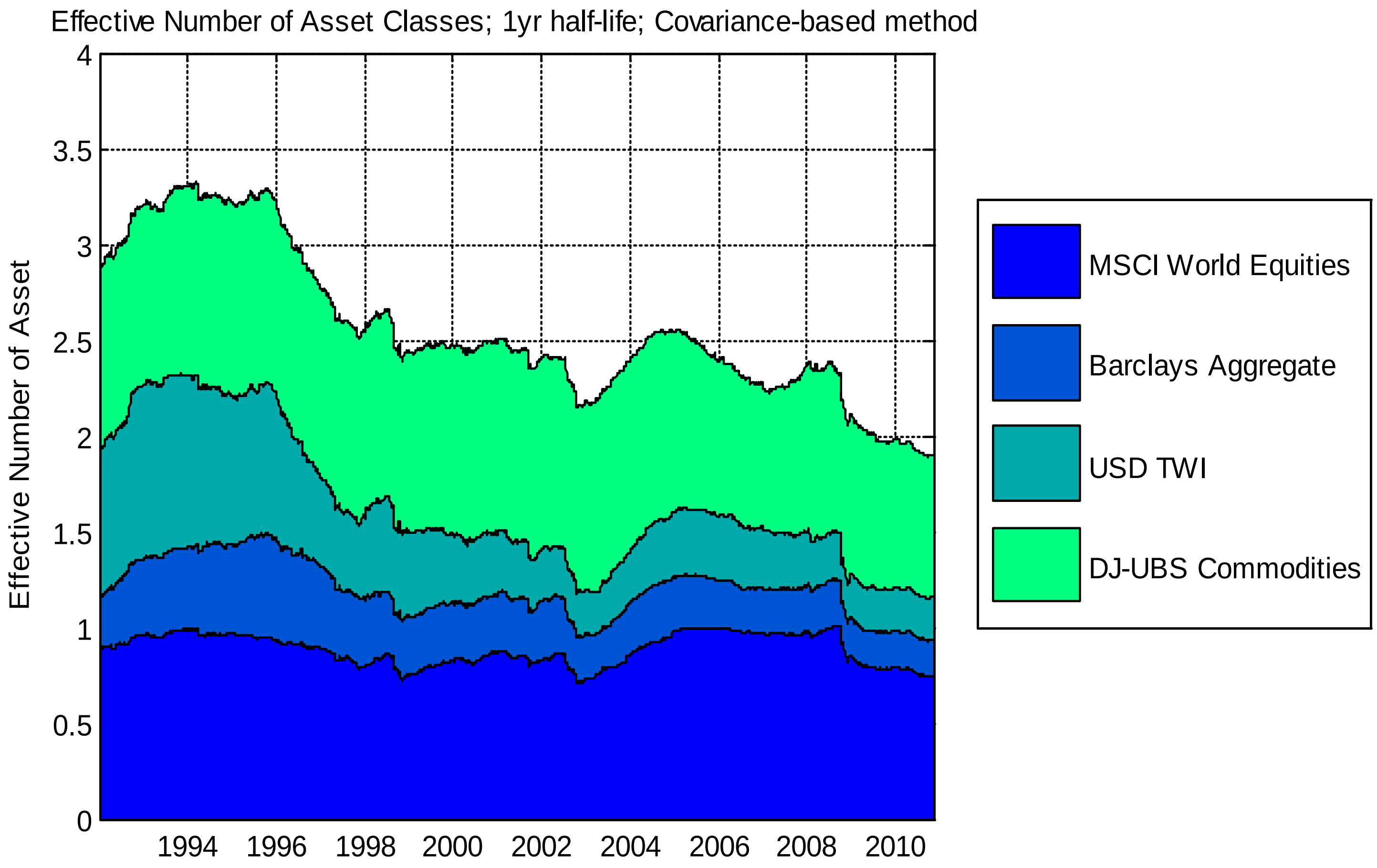
Diversification Measures (cont)

- In order to compare our measures, adjust some of them:
  - Adjusted condition number: N * (inverse condition number)
  - Adjusted Herfindahl-Hirschman Index (HHI): inverse HHI
  - Adjusted absorption ratio: inverse absorption ratio
  - Adjusted average absolute correlation: $N * (1 - \rho)$

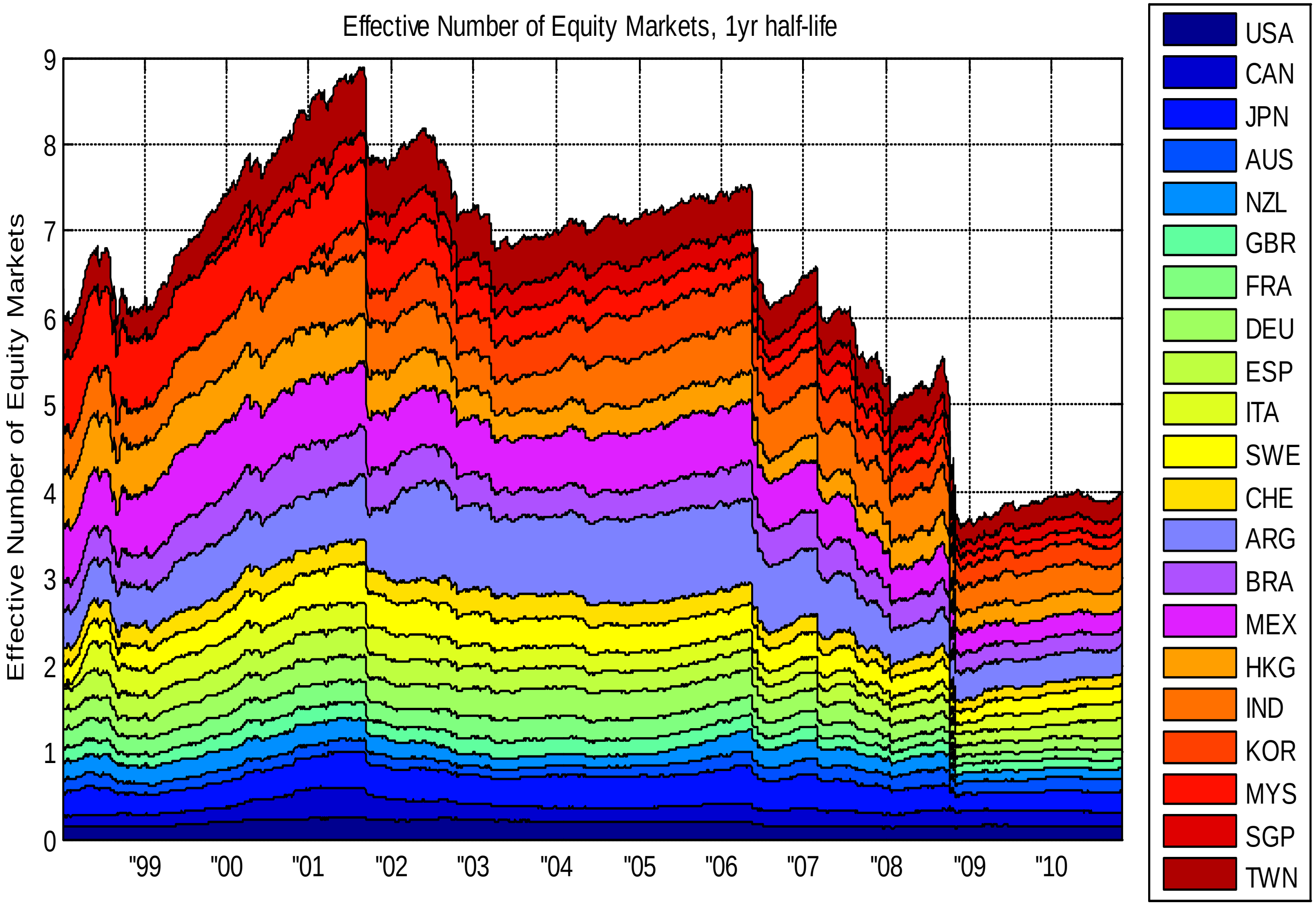
Diversification Measures (cont)



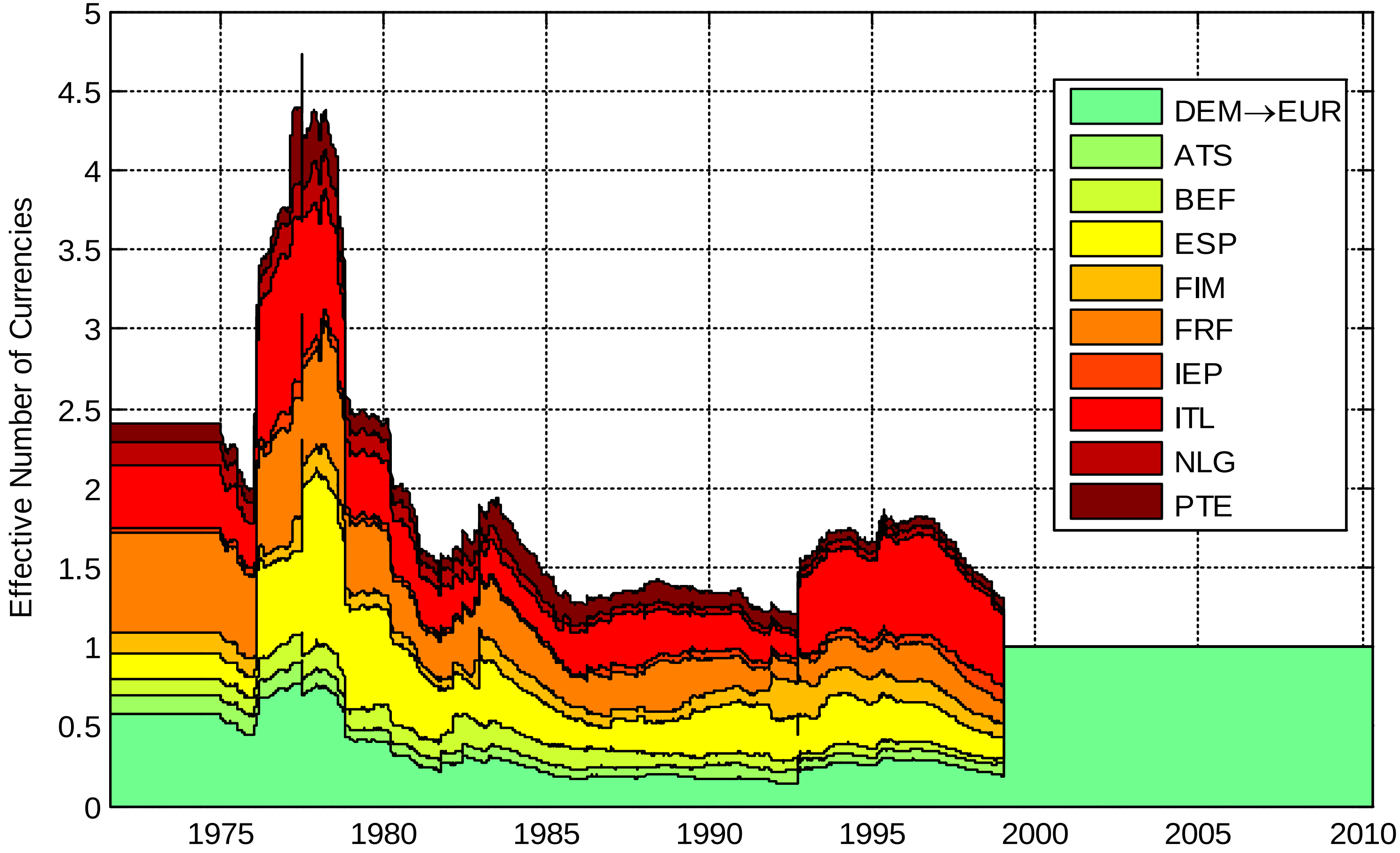
Example: Global Asset Classes



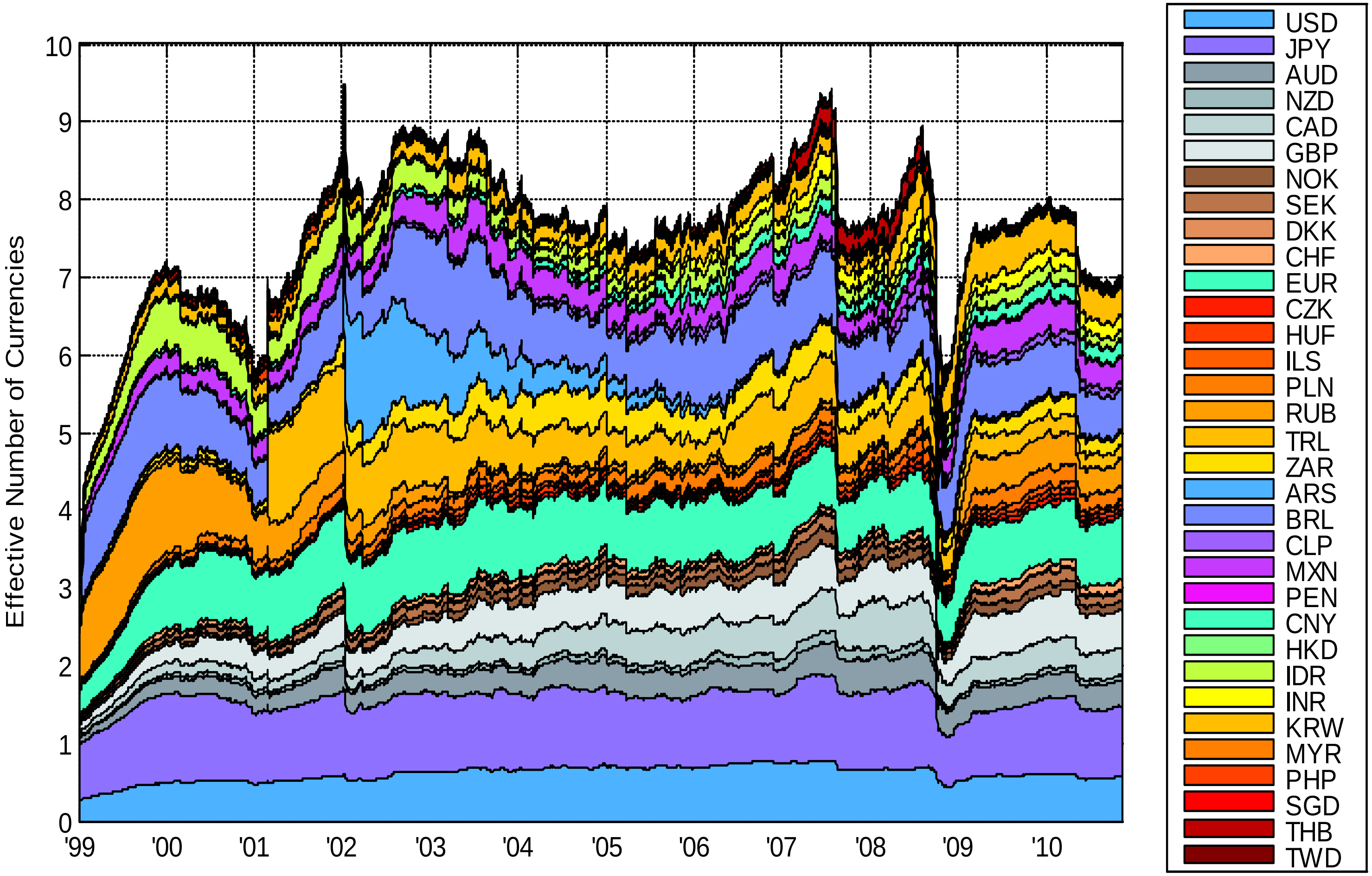
Example: Equity Markets Since 1998



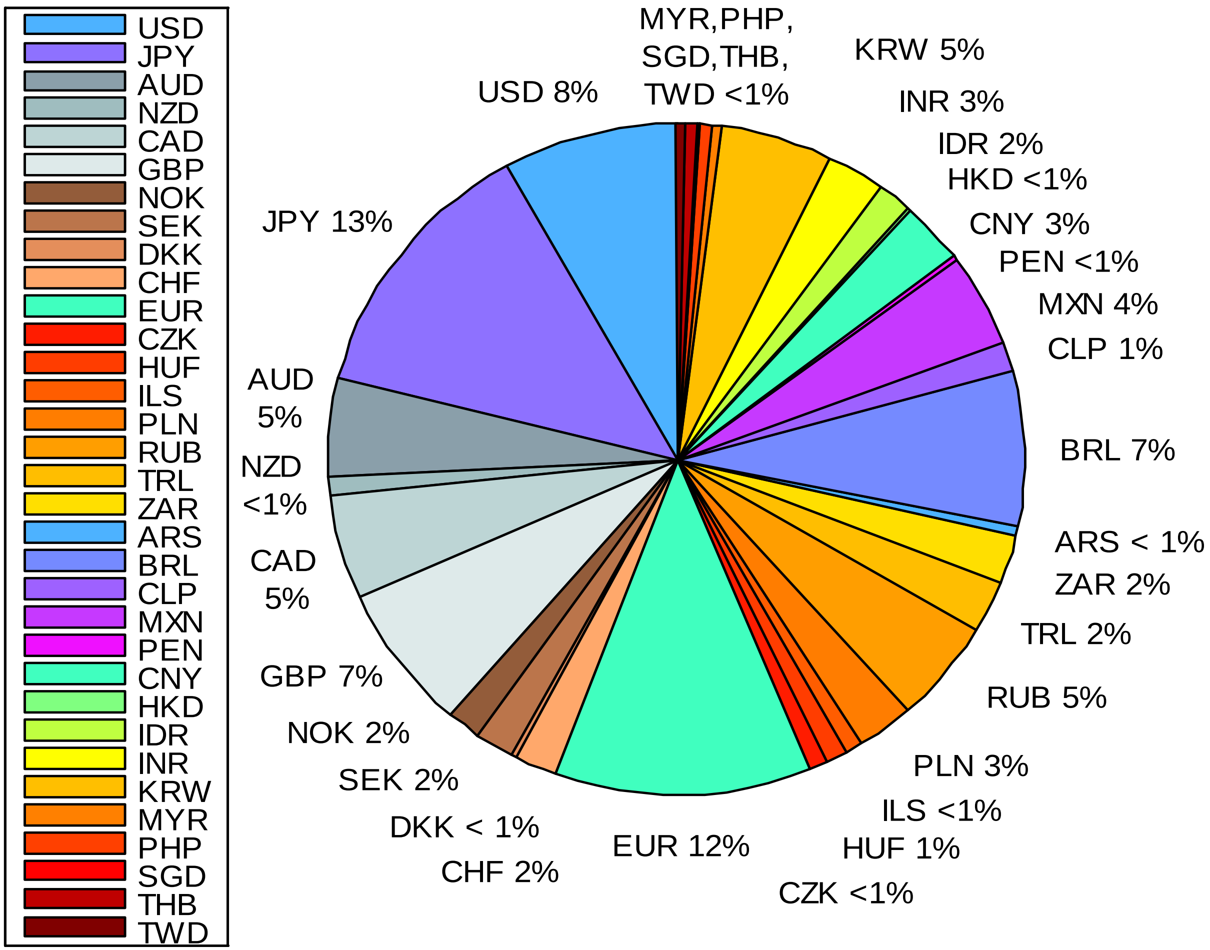
Example: Euro convergence, USD



Example: Currencies



Currency pie chart



Grinold-Kahn Framework

- Fundamental Law of Active Management

$$IR = IC * \sqrt{BR}$$

- What's the strategy's effective breadth?
- It can't be more than the effective number of assets
- If IC is the same, IR declines in concentrated markets
- In a very concentrated market, the breadth is close to 1
- The manager can only be "long" or "short," market timing

Optimization in Concentrated Markets

- Problem with classical MVO: the covariance matrix is ill-conditioned
  - error maximization or noise amplification
  - optimized weights bear little resemblance to alphas
- Widely used solution: shrink the covariance matrix to identity
- *Sharper Angle Optimization* employs robust optimization techniques to shrink the covariance matrix dynamically

  http://ssrn.com/abstract=1483412 (MG and Greg Jones, 2009)

Portfolio Construction and Risk

- Optimization: start with views, get portfolio weights
- Goal: maximize return while controlling risk
- What is “risk”?
  - Volatility, VAR, CVAR, drawdowns, skewness, liquidity, **leverage**
- Leverage is risky, we propose controlling it in 3 steps
  - **Inputs:** control the leverage of inputs
  - **Direction:** control the direction of the weights vector, limit leverage of noise alphas
  - **Magnitude:** control the overall portfolio leverage
- *Sharper Angle Optimization* deals with the *direction* piece of this puzzle

Angle Between Alphas and Weights

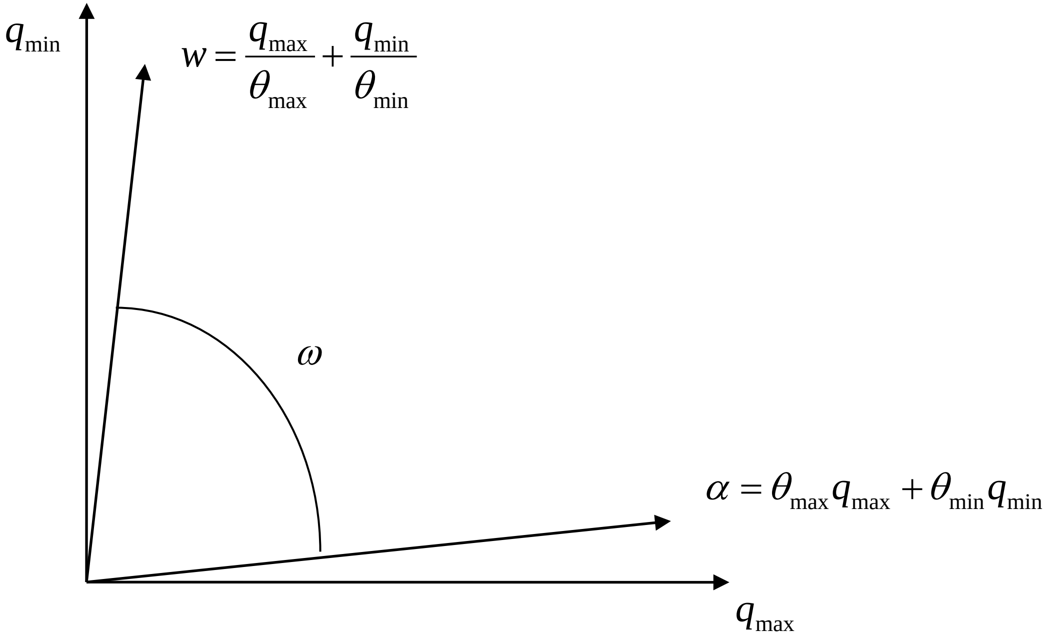
- Quantify the difference between alphas α and weights w by looking at the *angle* ω

$$\cos(\omega) = \frac{\alpha' w}{\sqrt{\alpha'\alpha}\sqrt{w'w}}$$

Ill-conditioned matrices

- For very ill-conditioned covariance matrices, alphas and weights could be close to orthogonal

q_{min}, q_{max} are eigenvectors with $\theta_{min}, \theta_{max}$ eigenvalues



Easy Remedy: Shrink to Identity

- Popular technique: consider following family

$$\Sigma(t) = t\frac{\mathrm{tr}\Sigma}{N} I_N + (1-t)\Sigma$$

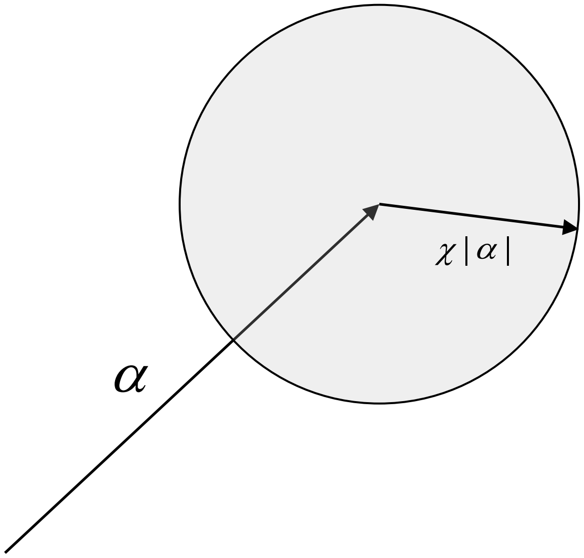
  of matrices "shrinking" Σ to the identity matrix

- <u>Practical question: what's t, or how much do you shrink?</u>

- Our answer: shrink enough for the degeneracy number to guarantee that the angle is not more than say, 60 degrees.

Robust Optimization: Spherical Cloud

- Alphas and covariances have estimation errors, or "uncertainty clouds"



- We assume that the uncertainty cloud U_{α} is a sphere centered at α with the radius χ|α|

Robust Optimization: Setup

- Classical optimization problem

$$\max_{w} w'\alpha \quad \text{subject to } w'\Sigma w \le \sigma_0^2$$

- Robust optimization problem

$$\max_{w} \min_{U_\alpha} w'\alpha \quad \text{subject to} \quad w'\Sigma w \le \sigma_0^2$$

Robust Optimization and the Angle

- After a bit of algebra, we get:

$$\Sigma(\chi,\alpha) = \chi I_N + \psi(\chi,\alpha)\Sigma$$

  so the covariance matrix Σ is conditional on α

- Robust optimization guarantees that

$$\cos(\omega) \geq \chi$$

- Shrinks conditional on a given (χ,α)

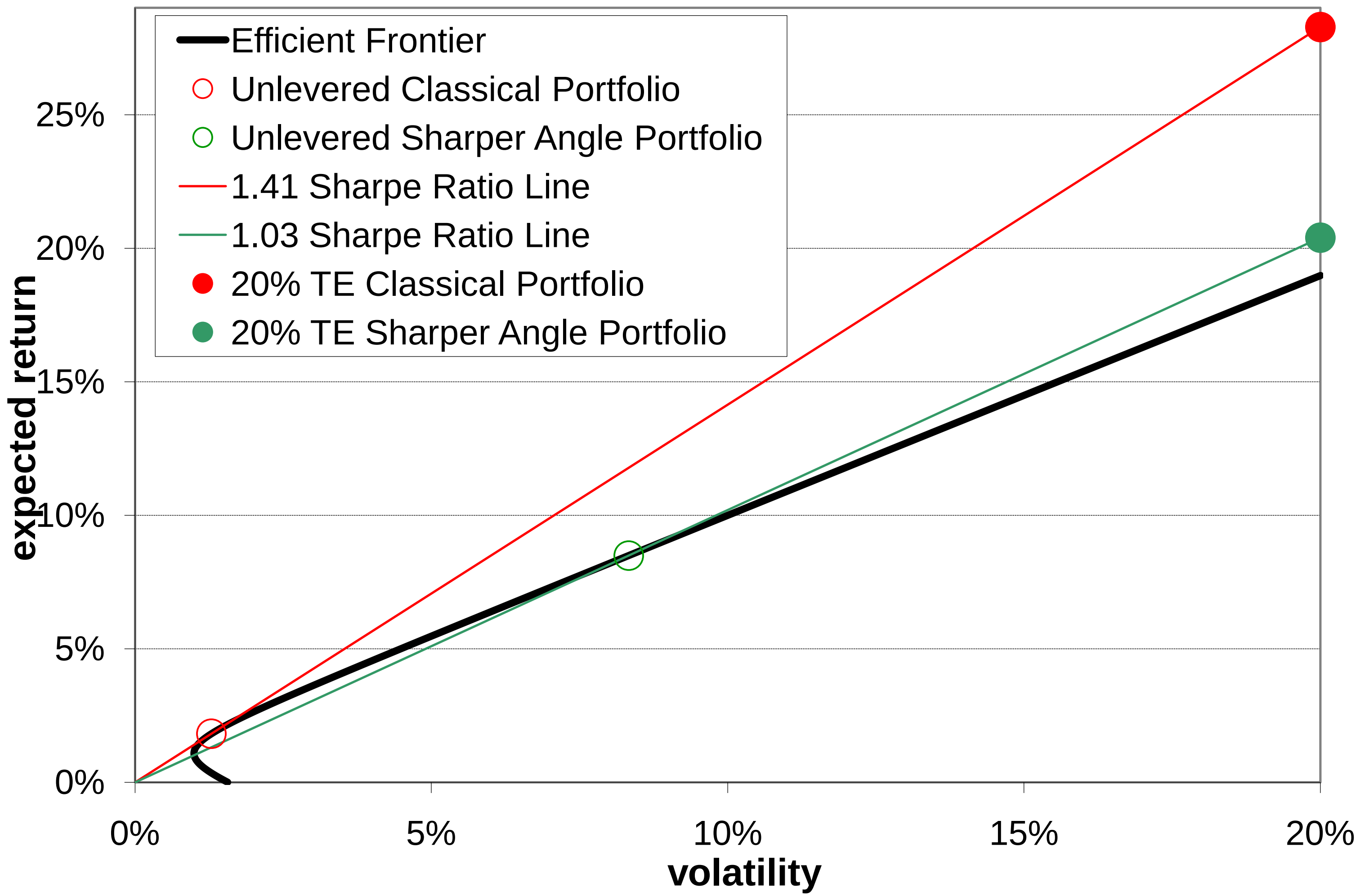
Sharper Angle Portfolio: 2D

- Consider a 2-dimensional example
- Compare Mean-Variance (MV) and Sharper Angle (SA) optimized weights

	alpha	volatility	MV weight	SA weight
asset 1	10%	10%	141%	200%
asset 2	1%	1%	1414%	40%

- Sharper Angle Optimization levers Asset 2 and the whole portfolio to a much lesser degree

Efficient Frontier



Concentrated Markets and Liquidity

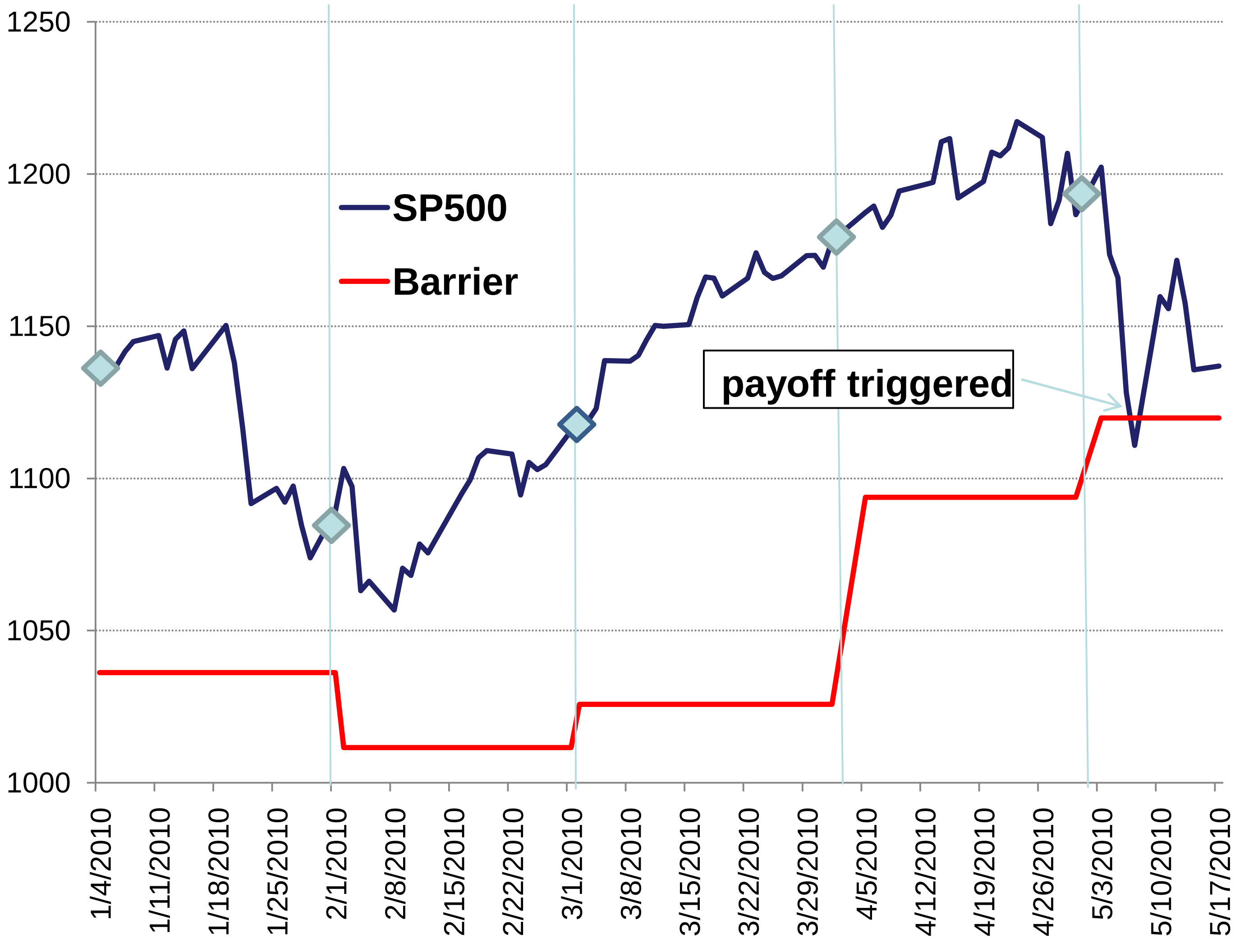
- "Everyone is trying to get out through a small door at the same time"
- Illiquid asset is purchased at a discount to its "fundamental value"
- Buying illiquid asset = writing "option to trade"
- The illiquidity discount = price of the liquidity option
- A few models pricing these liquidity options
- In particular, Golts-Kritzman model (2010)
- It is applicable to a wide variety of assets/investments

  http://ssrn.com/abstract=1584857

Liquidity Options

- Fundamentals are properties the *asset/investment*
- Liquidity is specific to the *market player*
- Price depends on three things:
  - volatility of an observable *reference process*
  - time to expiry
  - length of the *liquidity interval*
- The price of liquidity option increases
  - with increasing volatility
  - with increasing time to expiry
  - with decreasing length of the *liquidity interval*

Liquidity Options: Example



Conclusion

- Entropy-based <u>*effective number*</u> of assets is an intuitive way to measure diversification consistent with Grinold-Kahn
- Sharper Angle Optimization (and robust optimization techniques in general) produce better portfolios in concentrated markets
- The optionality of liquidity should be much better understood and modeled, especially in the presently highly concentrated markets