# Performance attribution for multi-layered investment decisions

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# The attribution problem

- Often investments involve multiple layers of decisions
- For example, in asset allocation the weight in each asset class is usually a result of several decisions
- For instance, the allocation to US small cap value equity depends on how much we allocate to:
  - equity vs fixed income,
  - US equity vs international equity,
  - large cap vs small cap, and
  - value vs growth
- The goal is to measure the impact of each allocation decision

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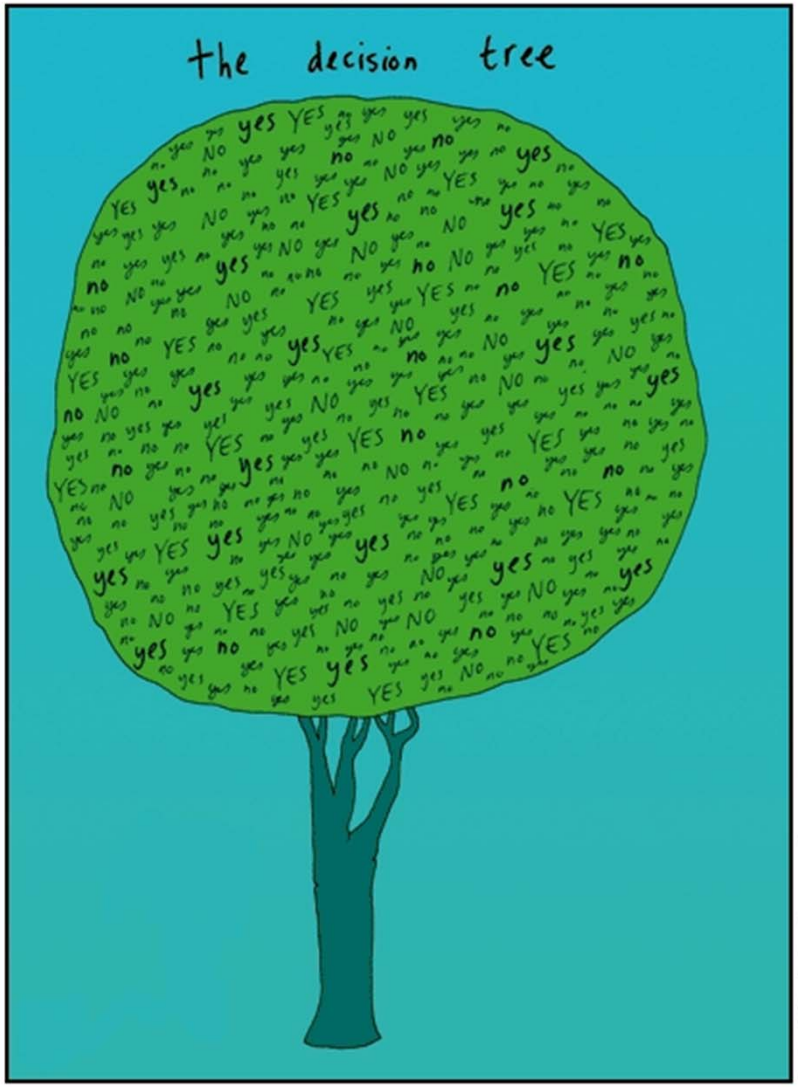
# Solution

- The proposed approach disentangles layers of investment decisions and measures impact of each decision separately
- It is most suited for performance attribution in asset allocation but can also be used in other areas such equity and fixed income
- The algorithm is very simple and can be easily implemented for arbitrary decision trees



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# Algorithm

# Total active return by asset groups

Total active return of a portfolio is equal to:

$$R_{Portfolio} - R_{Benchmark} = \sum_i w_i R_i - \sum_i w_i' R_i = \sum_i R_i \left( w_i - w_i' \right) \qquad (1)$$

where $w_i$ = portfolio weight of asset i

$w_i'$ = benchmark weight of asset i

$R_i$ = return of asset i

It can be divided into active returns of major groups of assets. For example, active return of equity (group $G_1$) and fixed income (group $G_2$):

$$(1) = \sum_{i \in G_1} R_i \left( w_i - w_i' \right) + \sum_{i \in G_2} R_i \left( w_i - w_i' \right) \qquad (2)$$

# The key formulas

Next, let's break down the active return of each group into two components:

$$\sum_{i \in G_k} R_i \left( w_i - w_i' \right)$$

Active return of the group

$$(3) \quad \sum_{i \in G_k} R_i \left( w_i - w_i' \frac{\sum_{i \in G_k} w_i}{\sum_{i \in G_k} w_i'} \right) \quad + \quad \sum_{i \in G_k} R_i w_i' \left( \frac{\sum_{i \in G_k} w_i}{\sum_{i \in G_k} w_i'} - 1 \right) \quad (4)$$

Impact of decisions within the group

Impact of under/overweight of the group itself

# Impact of decisions within the group

- In (3), we adjust benchmark weights by ratio: $\frac{\text{portfolio weight of the group}}{\text{benchmark weight of the group}}$

  - In effect, we adjust the benchmark weight of the group to the portfolio level
  - This removes the impact of the group under/overweight
  - Meanwhile we preserve the proportions of the weights within the group for both the portfolio and the benchmark
  - This allows us to measure the impact of decisions within the group

$$(3) \qquad \sum_{i \in G_k} R_i \left( w_i - w_i' \frac{\sum_{i \in G_k} w_i}{\sum_{i \in G_k} w_i'} \right)$$

Preserve proportions

Adjust benchmark weights

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# Impact of under/overweight of the group itself

- In (4), we use the benchmark return of the group and multiply it by the group under/overweight
  - Thus, we use the benchmark composition of the group and
  - Measure the pure impact of the group under/overweight

$$\sum_{i \in G_k} R_i w_i' \left( \frac{\sum_{i \in G_k} w_i}{\sum_{i \in G_k} w_i'} - 1 \right) \qquad (4)$$

Use benchmark composition

Multiply by group under/overweight

# Combining group impacts

To continue with the example of two groups, let's combine the impacts of group under/overweights to measure impact of equity vs fixed income decision

$$(2) = \sum_{i \in G_1} R_i \left( w_i - w_i' \frac{\sum_{i \in G_1} w_i}{\sum_{i \in G_1} w_i'} \right) + \sum_{i \in G_1} R_i w_i' \left( \frac{\sum_{i \in G_1} w_i}{\sum_{i \in G_1} w_i'} - 1 \right) +$$

$$+ \sum_{i \in G_2} R_i \left( w_i - w_i' \frac{\sum_{i \in G_2} w_i}{\sum_{i \in G_2} w_i'} \right) + \sum_{i \in G_2} R_i w_i' \left( \frac{\sum_{i \in G_2} w_i}{\sum_{i \in G_2} w_i'} - 1 \right)$$

Impact of equity vs fixed income decision

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# Further steps

- We leave the second component of each group (impact of under/overweight of the group itself) as is

- This will be the final component of the attribution

- We continue dividing the first component of each group (impact of decisions within the group) in the same way as we divided the total active return of the portfolio

# Dividing into subgroups

Denote $w_i' \dfrac{\sum_{i \in G_k} w_i}{\sum_{i \in G_k} w_i'}$ by $w_i''$, then the first component is equal to:

$$(3) = \sum_{i \in G_k} R_i \left( w_i - w_i' \frac{\sum_{i \in G_k} w_i}{\sum_{i \in G_k} w_i'} \right) = \sum_{i \in G_k} R_i \left( w_i - w_i'' \right)$$

which is the same as formula (1) and it can be similarly divided into subgroups and decomposed into formulas (3) and (4)

# Reaching the most granular level

We keep dividing groups into finer and finer subgroups until we reach the most granular level, i.e. when a group consists of a single asset

At the most granular level, the first component (impact of decisions within the group – formula (3)) is equal to zero:

$$\sum_{i \in G_k} R_i \left( w_i - w_i'''' \frac{\sum_{i \in G_k} w_i}{\sum_{i \in G_k} w_i''''} \right) = R_i \left( w_i - w_i'''' \frac{w_i}{w_i''''} \right) = 0$$

And the entire active return of the group (single asset) is equal to the second component (impact of its under/overweight – formula (4))

$$\sum_{i \in G_k} R_i w_i'''' \left( \frac{\sum_{i \in G_k} w_i}{\sum_{i \in G_k} w_i''''} - 1 \right) = R_i w_i'''' \left( \frac{w_i}{w_i''''} - 1 \right) = R_i (w_i - w_i'''')$$

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# Three asset example

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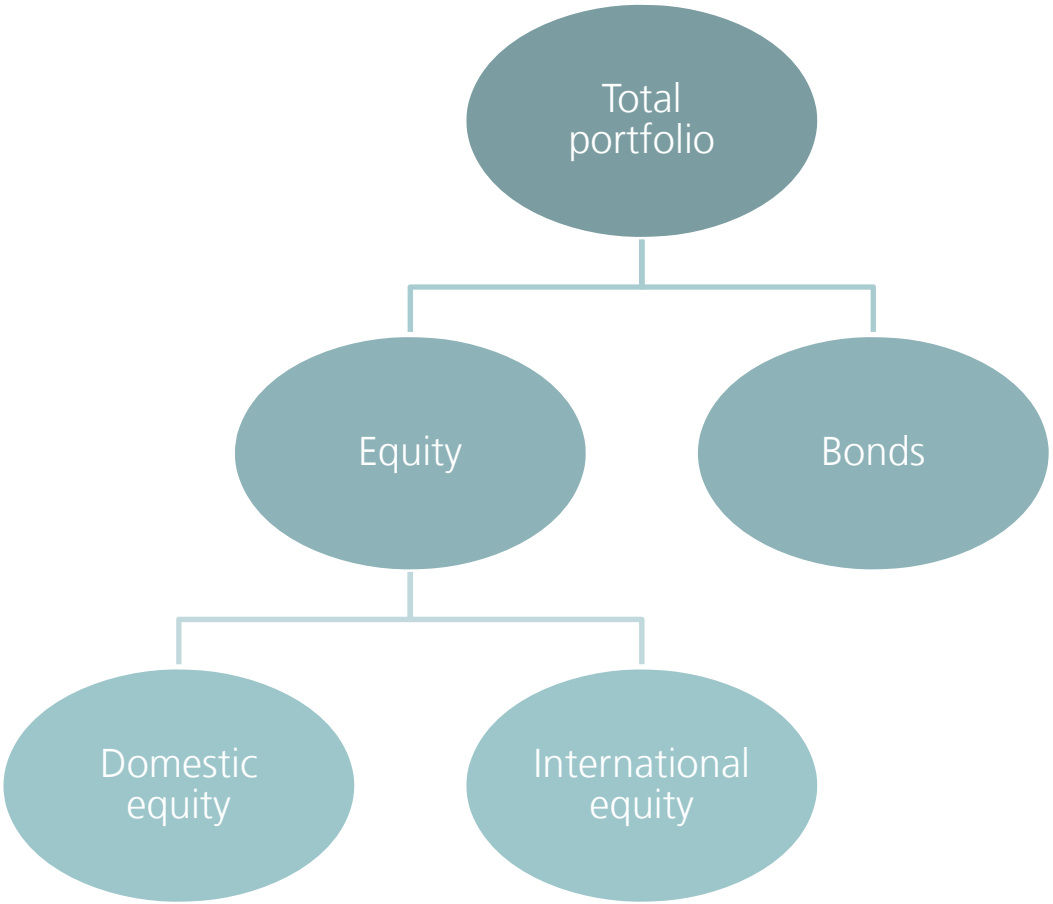
# Multiple layers of investment decisions



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# Simplistic approach

| Asset | Portfolio weight | Benchmark weight | Active weight | Return | Impact (active weight x return) |
|---|---|---|---|---|---|
| Domestic equity | 35% | 40% | -5% | 7% | -0.35% |
| Int. equity | 25% | 10% | 15% | 8% | 1.2% |
| Bonds | 40% | 50% | -10% | 4% | -0.4% |
| **Total** | **100%** | **100%** | **0%** | | **0.45%** |

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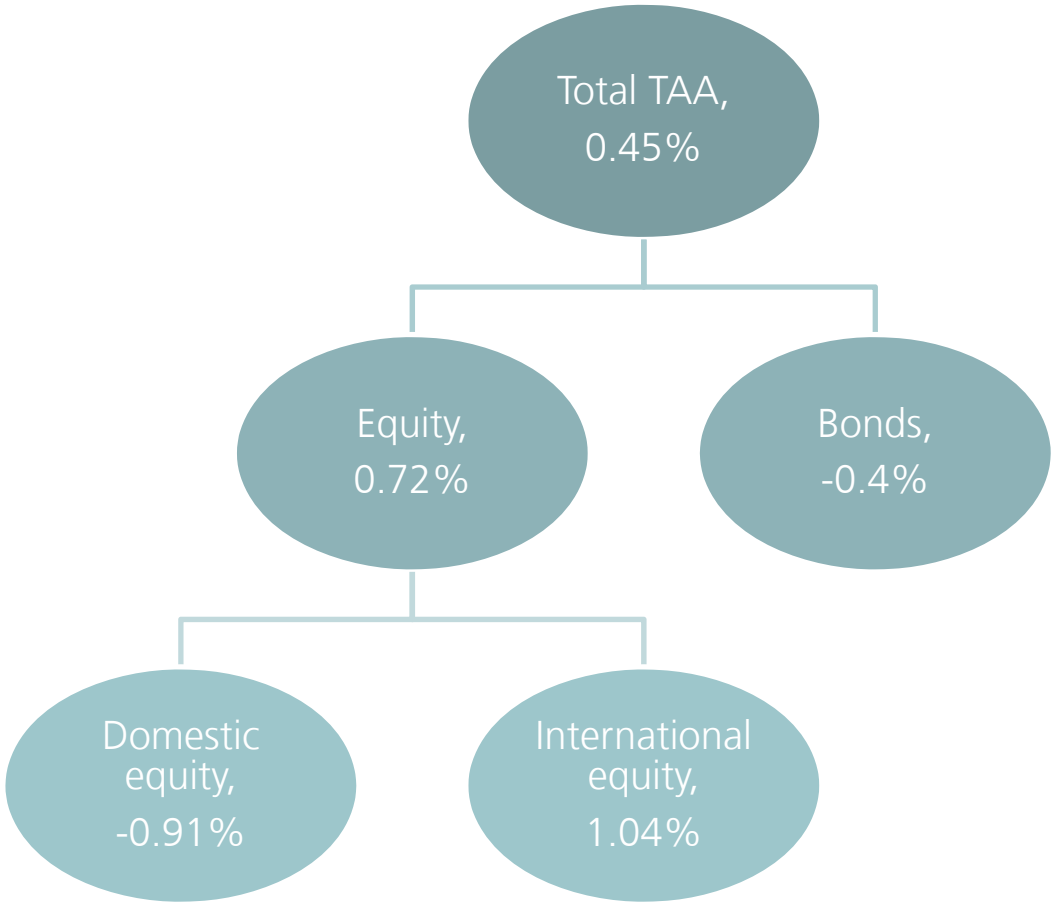
# Proposed approach

Equity/bond decision, 0.32%

Decisions within equity, 0.13%



# Dividing into major groups

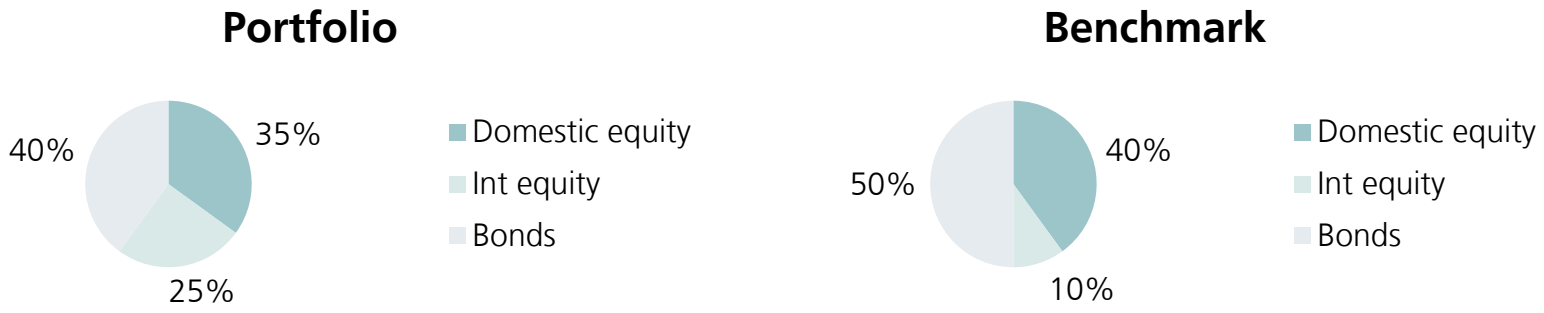
We have two groups at the first level in this example: equity and bonds

Equity group active return (formula (2)) is equal to:

$$\sum_{i \in G_{Equity}} R_i \left( w_i - w_i' \right) = 7\% \cdot (35\% - 40\%) + 8\% \cdot (25\% - 10\%)$$

Bond group active return (formula (2)) is equal to:

$$\sum_{i \in G_{Bonds}} R_i \left( w_i - w_i' \right) = 4\% \cdot (40\% - 50\%)$$



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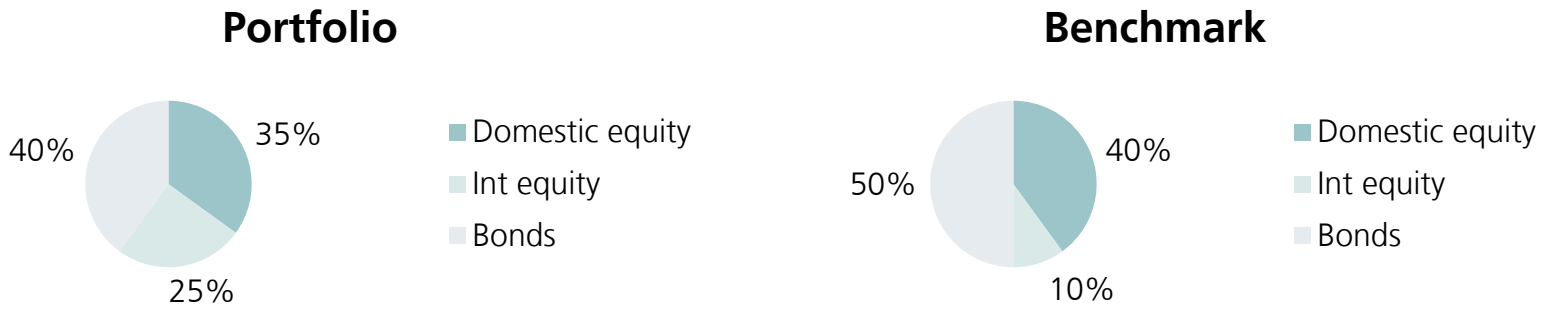
# Measuring impact of decisions within groups

For equity, the impact of decisions within the group (formula (3)) is equal to:

$$\sum_{i \in G_{Equity}} R_i \left( w_i - w_i' \frac{\sum_{i \in G_{Equity}} w_i}{\sum_{i \in G_{Equity}} w_i'} \right) = 7\% \cdot \left( 35\% - 40\% \cdot \frac{60\%}{50\%} \right) + 8\% \cdot \left( 25\% - 10\% \cdot \frac{60\%}{50\%} \right) =$$

$$= 7\% \cdot (35\% - 48\%) + 8\% \cdot (25\% - 12\%) = -0.91\% + 1.04\% = 0.13\%$$



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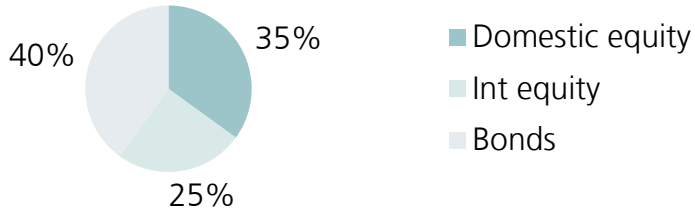
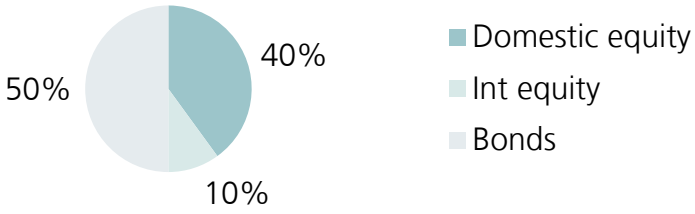
# Weights to measure impact of decisions within groups

## Original weights

**Portfolio**

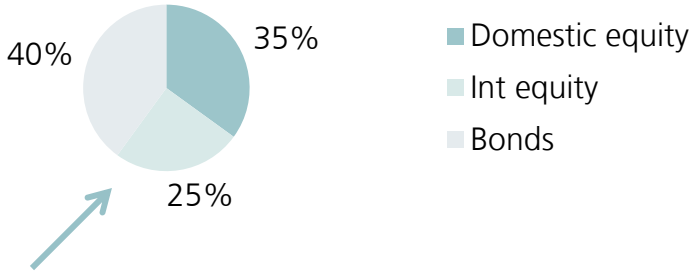



**Benchmark**



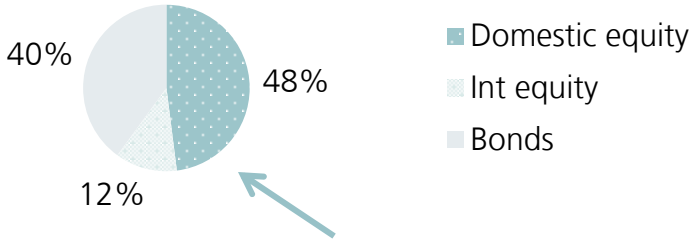

## Weights after the benchmark adjustment (formula (3)):

**Portfolio**




Same as original

**Benchmark**




Adjust bond and equity weights to portfolio level;
Preserve the proportion within equity

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# Measuring impact of decisions within groups

For bonds, the impact of decisions within the group (formula (3)) is equal to zero, as there is just one asset in the group:

$$\sum_{i \in G_{Bonds}} R_i \left( w_i - w_i' \frac{\sum_{i \in G_{Bonds}} w_i}{\sum_{i \in G_{Bonds}} w_i'} \right) = 4\% \cdot \left( 40\% - 50\% \cdot \frac{40\%}{50\%} \right) = 4\% \cdot (40\% - 40\%) = 0\%$$

# Measuring impact of group under/overweight

For equity, the impact of the group under/overweight (formula (4)) is equal to:

$$\sum_{i \in G_{Equity}} R_i w_i' \left( \frac{\sum_{i \in G_{Equity}} w_i}{\sum_{i \in G_{Equity}} w_i'} - 1 \right) = 7\% \cdot 40\% \cdot \left( \frac{60\%}{50\%} - 1 \right) + 8\% \cdot 10\% \cdot \left( \frac{60\%}{50\%} - 1 \right) =$$

$$= \left( 7\% \cdot 40\% + 8\% \cdot 10\% \right) \cdot \left( \frac{60\%}{50\%} - 1 \right) = 7\% \cdot 8\% + 8\% \cdot 2\% = 0.72\%$$

For bonds, the impact of the group under/overweight (formula (4)) is equal to:

$$\sum_{i \in G_{Bonds}} R_i w_i' \left( \frac{\sum_{i \in G_{Bonds}} w_i}{\sum_{i \in G_{Bonds}} w_i'} - 1 \right) = 4\% \cdot 50\% \cdot \left( \frac{40\%}{50\%} - 1 \right) = 4\% \cdot (-10\%) = -0.4\%$$

The overall impact of the equity/bond decision is equal to:

$$0.72\% - 0.4\% = 0.32\%$$

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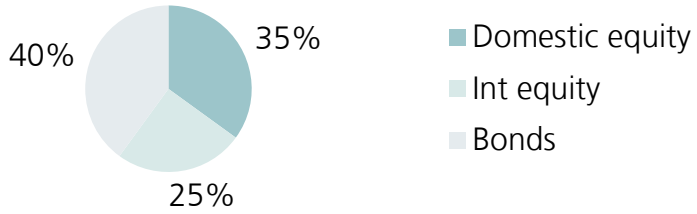
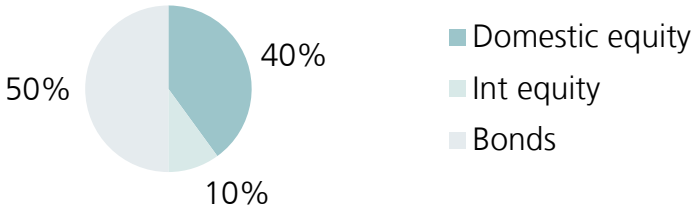
# Weights to measure impact of group under/overweight

## Original weights

**Portfolio**

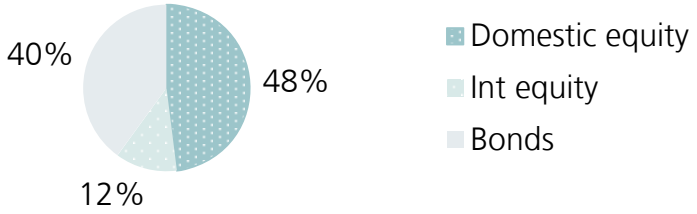



**Benchmark**



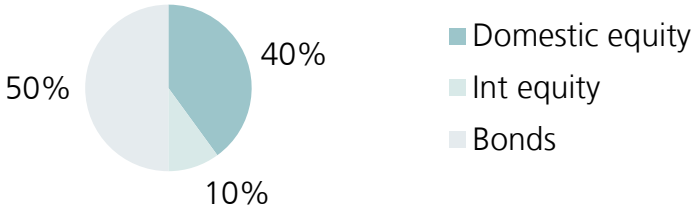

## Weights after using benchmark proportions in the portfolio (formula (4)):

**Portfolio**




Keep bond and overall equity weights as original;
Use the benchmark proportions within equity

**Benchmark**




Same as original

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