

Evaluating the Effects of Investment Decisions on Risk-Adjusted Returns

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For QWAFEFW's

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Decomposition

- Finance's standard approach to ex-post evaluation of causation:
 - If there are 2 causes of 7, then they must have values 5 & 2 since $7 = 5+2$.
- Leads to problems.
- Examples:
 - Performance Attribution (Multi Decision Layered)
 - Risk Attribution (Menchero&Hu)(Bertrand)
 - Also in Trade Returns and Contributions

Decomposition = Top-Down modeling

1. Portfolio-level (Total)

Impose economic requirements
that completely specify.

2. Component-level (Issue or Decision)

Rely upon arbitrary assumptions
& then Partially restrict them.

Decomposing Active Return for Multiple Kinds, k , of Decisions for Attribution

$$\Delta R \equiv R^P - R^B = \sim = \text{All} + \text{Sel} + \text{Int} \equiv \sum_k X_k$$

Impose Portfolio-level Definition of ΔR

And formal requirement on sum of attributes

And, perhaps, intuitive monotonicity of

- All w/over-weighting good sectors and
- Sel w/over-weighting good issues in sectors

Decomposing Active Return with freedom

$$\begin{aligned}\Delta R &\equiv R^P - R^B = \text{All}' + \text{Sel}' + \text{Int}' \equiv \sum_k X'_k \\ &= (\text{All} + \mathbf{Ae}) + (\text{Sel} + \mathbf{Se}) + (\text{Int} + \mathbf{Ie})\end{aligned}$$

where $\mathbf{Ae} + \mathbf{Se} + \mathbf{Ie} = 0$. Thus, \mathbf{X}_k **arbitrary!**

Can even still save intuitive monotonicity of

- All' w/over-weighting good sectors and
- Sel' w/over-weighting good issues in sectors

By fully imposing Meaning
Only at the Portfolio level,
& then restrictions below,
We do not get valid meaning
at the component (attribute) level.

Decomposing Risk-Adjusted Returns

$$\begin{aligned}
 \text{IR} &\equiv \frac{\langle \Delta R \rangle}{\sigma(\Delta R)} = \frac{\sum_k \langle X_k \rangle}{\sigma(\Delta R)} \\
 &= \frac{\sum_k [\rho(X_k, \Delta R) * \sigma(X_k)]}{\sigma(\Delta R)} * \frac{\langle X_k \rangle}{[\rho(X_k, \Delta R) * \sigma(X_k)]}.
 \end{aligned}$$

$$\equiv \sum_k \text{WR}_k * \text{IR}_k$$

where $\rho \equiv \text{Correlation} \rightarrow \sum_k \text{WR}_k = 1.$

Decomposing Risk-Adjusted Returns

- $$IR \equiv \frac{\langle \Delta R \rangle}{\sigma(\Delta R)} = \frac{\sum_k \langle X_k \rangle}{\sigma(\Delta R)}$$

$$= \frac{\sum_k [\rho(X_k, \Delta R) * \sigma(X_k) + E_k]}{\sigma(\Delta R)} * \frac{\langle X_k \rangle}{[\rho(X_k, \Delta R) * \sigma(X_k) + E_k]}$$

$$\equiv \sum_k WR'_k * IR'_k.$$

$$\sum_k E_k = 0 \rightarrow \sum_k WR'_k = 1.$$

But, since E_k is arbitrary, **IR'_k is Arbitrary!**

Decomposing Risk-Adjusted Returns

$$IR \equiv \frac{\langle \Delta R \rangle}{\sigma(\Delta R)} = \frac{\sum_k \langle X_k \rangle}{\sigma(\Delta R)} = \frac{\sum_k [\langle X_k \rangle + a a_k]}{\sigma(\Delta R)}$$

$$\equiv \sum_k IRCont_k, \quad \text{where} \quad \sum_k a a_k = 0.$$

$IRCont_k \equiv$ “The portion of IR attributed to decision k.”

But $IRCont_k$ is Arbitrary!

Note: All the decompositions of IR
have the same denominator, $\sigma(\Delta R)$.

Again,
By fully imposing Meaning
Only at the Portfolio level,
We do not get valid meaning
at the component (attribute) level.

Properties
(like returns and attributes)
need to be
economically defined
at the level they are employed,
else the arbitrariness breeds
nonsense.

HATS

Top-Down Modeling

The Decomposition of the ‘Hat Quantity’

What is the ‘cause’ of there being 14 hats between us?

HAT Number: #H_j

- HATS: You 11H & Me 3H
- Model: 14H = 9H + 5H
- Top-level meaning:
 - 1. Correctly Adds up: $9 + 5 = 14$
- Lower level, only formal:
 - 2. Correctly ordered: Y>M.
 - 3. Correct Parity: All odd.
 - 4. Correct Units: H.
- Still Wrong since not defined at atomic level: #H_j=?
- Decomposed Total H → Incorrectly model #H_j.

Decision Evaluation

- Attribution to **Controllables**:
Evaluate the effects of decisions,
as opposed to the effects of the market.
- Avoid arbitrariness
Define these effects
at the level of individual decisions.

Will here apply decision evaluation
to Risk Attribution.

Decision Process

- Choose investment process employed.
- EXAMPLE w/Simple Single-Period (multiple sub-periods):

- Null:
$$R^B = \sum_S W^B_S R^B_S$$

- Allocation:
$$\begin{aligned} R^A &= \sum_S W^P_S R^B_S \\ &= \sum_S W^P_S \sum_{j \in S} W^B_j R_j \end{aligned}$$

- Selection:
$$\begin{aligned} R^P &= \sum_S W^P_S R^P_S \\ &= \sum_S W^P_S \sum_{j \in S} W^P_j R_j \end{aligned}$$

- For now, Forgo the harder:
 - Compounded Multi-Period,
 - Nested
 - Forking Tree

DEFINE 'ATTRIBUTE'

The value of a controllable decision level attribute

≡ The effect on the parameter
of implementing the decision
within a set of multi-layered decisions ≡ **Difference** =

The value of the parameter (e.g. IR)

after implementing the decision

Minus

• The value of the parameter
before implementing the decision.

This def. vs. Standard PA for FI & multi: T, Curr, Asset, Nesting.

Decision Process

- EXAMPLE w/ days:

- Null: $R^B = \sum_S W^B_S R^B_S$

- Allocation: $R^A = \sum_S W^P_S R^B_S$

- Selection: $R^P = \sum_S W^P_S R^P_S$

Decision Evaluation Ex.

- Chose Information Ratio

- $$\text{IR}^{P,B} \equiv \frac{\langle \Delta R \rangle}{\sigma(\Delta R)} = \frac{\langle R^P - R^B \rangle}{\sigma(R^P - R^B)}$$

- Allocation:
$$\text{IR}^{\text{All}} \equiv \text{IR}^{A,B} - \text{IR}^{B,B} = \text{IR}^{A,B}$$

- Selection:
$$\text{IR}^{\text{Sel}} \equiv \text{IR}^{P,B} - \text{IR}^{A,B} = \text{IR}^{P,B} - \text{IR}^{A,B}$$

Total:

$$\text{IR}^{P,B}$$

Decision Risk

The decision

To select holdings within sectors

On each of the days in the period

Increased the portfolio's IR by

$$IR^{Sel} = IR - IR^{A,B} = \frac{\langle R^P - R^B \rangle}{\sigma(R^P - R^B)} - \frac{\langle R^A - R^B \rangle}{\sigma(R^A - R^B)}$$

= The amount of IR attributed to Selection.

Decision Evaluation V. Decomposition Cont.

$$IR^{Sel} = \frac{\langle R^P - R^B \rangle}{\sigma(R^P - R^B)} - \frac{\langle R^A - R^B \rangle}{\sigma(R^A - R^B)} = \text{Effect of Sel on IR.}$$

Answers the precise Economic question re **difference**:
How much did the implementation of ‘Sel’ decision
increase the information ratio?

Whereas Decomposition’s

$$IR_Cont_A = \frac{\langle R^P - R^A \rangle}{\sigma(R^P - R^B)} = \frac{\langle R^P - R^B \rangle}{\sigma(R^P - R^B)} - \frac{\langle R^A - R^B \rangle}{\sigma(R^P - R^B)}$$

Is an economically ill-formed “measure”

Constructed by an arbitrary formal process and
then Named “Contribution of Selection to IR.”

Misinformation from Standard Decomposition

If R^P tracked R^B

But R^A did not track R^B at all,

Then can have cases where

Standard Decomposition results imply that

Selection does not contribute significantly to IR

When in fact, it contributes a lot.

And Decision Evaluation precisely shows this.

IR Attribution Example

- Consider,
 - a BM, B,
 - a state of the Fund after Allocation, A,
 - and a final position of the Fund, P. $e \ll 1$

- The returns over 4 periods are:

B:	-1/2	1/2	1/2	-1/2
A	3/2	1/2	1/2	3/2
P	3/2+e	5/2-e	5/2-e	3/2+e

IR Attribution Ex. Results

A-B	2	0	0	2
	$\langle A-B \rangle = 1$ & $SD(A-B) = 1$			
P-B	$2+e$	$2-e$	$2-e$	$2+e$
	$\langle P-B \rangle = 2$ & $SD(P-B) = e$			

- **Decomposition:**

$e=0.1$

Allocation = $\langle A-B \rangle / SD(P-B) = 1/e$ Medium 10

Selection = $\langle P-B \rangle / SD(P-B) - \langle A-B \rangle / SD(P-B) = 2/e - 1/e$ **Medium** 10

- **FDE:**

Allocation = $\langle A-B \rangle / SD(A-B) = 1/1 = 1$ Small 1

Selection = $\langle P-B \rangle / SD(P-B) - \langle A-B \rangle / SD(A-B) = 2/e - 1/1$ **Large** 19

IRR Attribution Ex's Conclusion

- Allocation → fund w/ gain & wild fluctuations.
- Selection → fund w/ same additional gain
& decreased fluctuations.

- Thus, Allocation → small IR,
- While **Selection** → **large** IR.

- Decomposition → wrong.
- **FDE** → **exactly right.**

Summary

- Standard methods of Decomposition
 - Define at the portfolio level,
Interpret at the decision or issue level.
 - Leading to formally consistent but misleading results.
- Decision Evaluation
 - Provides uniquely appropriate answers
to precisely meaningful economic questions
at the level sought.

Warning!

Standard Decomposition Methods can Dangerously Misguide

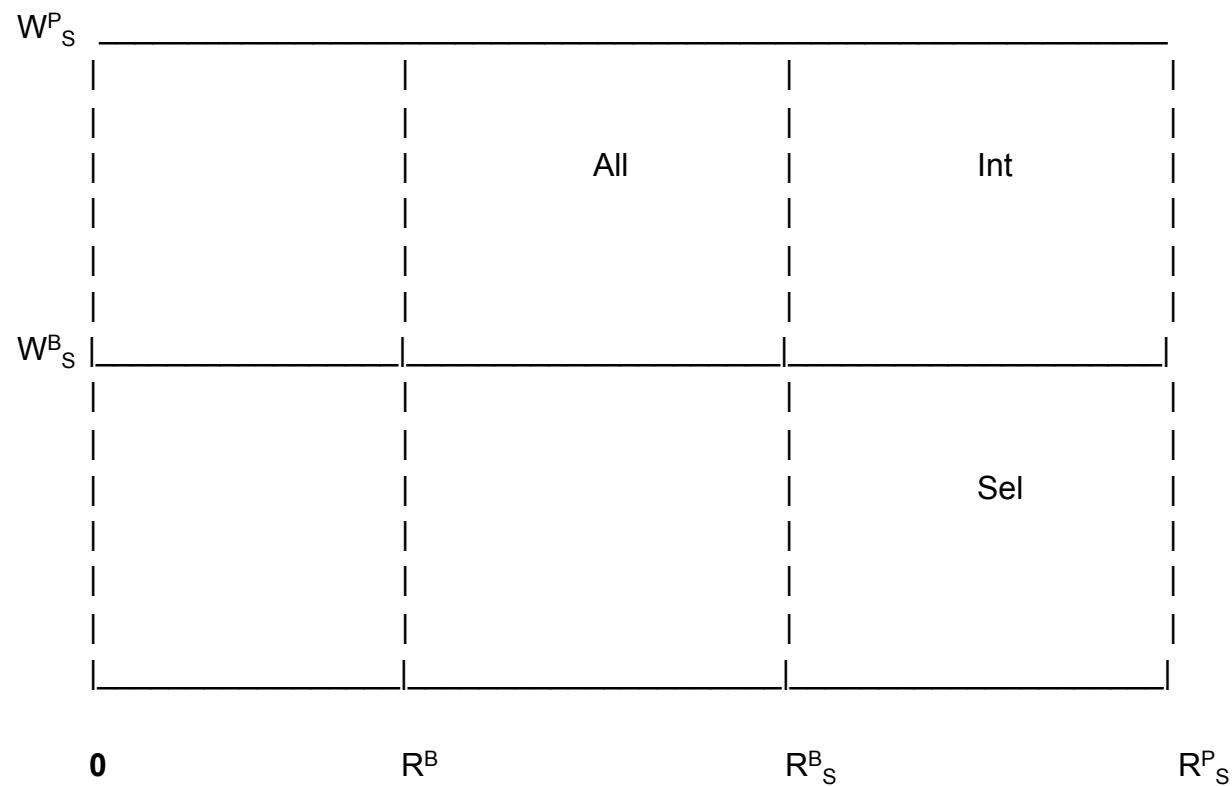
any Investment Processed that relies on knowing:

- Performance Attribution of Brinson-type, or
- Risk Attribution & Contribution
- OTHER SO DERIVED COMMON PARAMETERS,
 - Like component-level Trade inclusive returns & Contributions

Use Economically Meaningful Methods.

Decision Evaluation → Meaningful Results.

Attribute Definitions



Reference paper

<http://www.northinfo.com/documents/377.pdf>